Commun. Math. Phys. 114, 439–461 (1988)

Communications in Mathematical Physics

Smoothness of the Density of States in the Anderson Model at High Disorder

Anton Bovier, Massimo Campanino*, Abel Klein** and J. Fernando Perez***
Department of Mathematics, University of California, Irvine, CA 92717, USA

Abstract. We prove smoothness of the density of states in the Anderson model at high disorder for a class of potential distributions that include the uniform distribution.

1. Introduction

The Anderson model is given by the random Hamiltonian $H_\varepsilon = -\varepsilon/2\Delta + V$ on $l^2(\mathbf{Z}^d)$, where

$$(\Delta u)(x) = \sum_{y,|y-x|=1} u(y)$$

and $V(x)$, $x\in\mathbf{Z}^d$, are independent identically distributed random variables with common probability distribution μ. The characteristic function of μ will be denoted by h, i.e., $h(t)=\int e^{-itv}d\mu(v)$. The "disorder" is measured by $\varepsilon^{-1}, \varepsilon>0$.

If Λ is a finite subset of $\mathbf{Z}^d$, we will denote by $H_{\varepsilon,\Lambda}$ the operator H_ε restricted to $l^2(\Lambda)$ with zero boundary conditions outside Λ.

The integrated density of states, $N_\varepsilon(E)$, is defined by

$$N_\varepsilon(E) = \lim_{\Lambda\to\mathbf{Z}^d} |\Lambda|^{-1}\#\{\text{eigenvalues of } H_{\varepsilon,\Lambda} \leqq E\}.$$

It is a consequence of the ergodic theorem that for almost every potential the limit exists for all E and is independent of the potential [1–4]. $N_\varepsilon(E)$ is always a continuous function [5–7], being log-Hölder continuous under mild conditions [6].

In one-dimension a lot is known about the integrated density of states. Under mild conditions it is always Hölder continuous on compact intervals [8, 9] and under some minimal regularity assumptions on μ it is differentiable, even infinitely differentiable [10–12].

---

* Permanent address: Dipartimento di Matematica, II Universita di Roma, Via Orazio Raimondo, I-00173 (La Romanina) Roma, Italy

** Research partially supported by the NSF under grants DMS 83-01889 and INT 85-03418

*** Permanent address: Instituto de Fisica, Universidade de São Paulo, Caixa Postal 20516, São Paulo, S. P., Brazil. Research partially supported by the CNPq