

Is Every Approximate Trajectory of Some Process Near an Exact Trajectory of a Nearby Process?*

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Abstract. This paper deals with the problem “Can a noisy orbit be tracked by a real orbit?” In particular, we will study the one-parameter family of tent maps and the one-parameter family of quadratic maps. We write g_μ for either f_μ or F_μ with $f_\mu(x) = \mu x$ for $x \leq \frac{1}{2}$ and $f_\mu(x) = \mu(1 - x)$ for $x \geq \frac{1}{2}$, and $F_\mu(x) = \mu x(1 - x)$. For a given μ we will say: g_μ permits increased parameter shadowing if for each $\delta_x > 0$ there exists some $\delta_\mu > 0$ and some $\delta_f > 0$ such that every δ_f -pseudo g_μ -orbit starting in some invariant interval can be δ_x -shadowed by a real g_α -orbit with $\alpha = \mu + \delta_\mu$. We show that g_μ typically permits increased parameter shadowing.

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