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Stability of Shock Waves for the Broadwell Equations

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Abstract. For the Broadwell model of the nonlinear Boltzmann equation, there are shock profile solutions, i.e. smooth traveling waves that connect two equilibrium states. For weak shock waves, we prove asymptotic (in time) stability with respect to small perturbations of the initial data. Following the work of Liu [7] on shock wave stability for viscous conservation laws, the method consists of analyzing the solution as the sum of a shock wave, a diffusive wave, a linear hyperbolic wave and an error term. The diffusive and linear hyperbolic waves are approximate solutions of the fluid dynamic equations corresponding to the Broadwell model. The error term is estimated using a variation of the energy estimates of Kawashima and Matsumura [6] and the characteristic energy method of Liu [7].

1. Introduction

The Broadwell model for the nonlinear Boltzmann equation is

$$\begin{aligned}\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}\right)f_+ &= f_0^2-f_+f_-\,,\\ \frac{\partial}{\partial t}f_0 &= -\tfrac{1}{2}(f_0^2-f_+f_-)\,,\\ \left(\frac{\partial}{\partial t}-\frac{\partial}{\partial x}\right)f_- &= f_0^2-f_+f_-\,,\end{aligned}\tag{1.1}$$

in which f_+, f_0, f_- represent the densities of particles moving with speeds $1, 0, -1$ in the x direction. The physical significance of (1.1) is discussed in [2, 3]. Global existence for solutions of the initial value problem for (1.1) is proved in [10] and the fluid dynamic limit for (1.1) is analyzed in [3].

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