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Some Comments on the Sherrington-Kirkpatrick Model of Spin Glasses

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Abstract. In this paper the high-temperature phase of general mean-field spin glass models, including the Sherrington-Kirkpatrick (SK) model, is analyzed. The free energy in zero magnetic field is calculated explicitly for the SK model, and uniform bounds on quenched susceptibilities are established. It is also shown that, at high temperatures, mean-field spin glasses are limits of short-range spin glasses, as the range of the interactions tends to infinity.

1. Introduction

In this note we comment on the high-temperature properties of a class of mean-field spin glass models, including the Sherrington-Kirkpatrick (SK) model. Our method of analysis is the one developed in [1]. In [2], Aizenman et al. present a detailed analysis of the high-temperature behaviour of the SK model in zero magnetic field[1]. They also prove some results on the behaviour of the free energy at low temperatures and give bounds on the ground state energy density. While their results prove that the SK model exhibits a phase transition in zero magnetic field, as the temperature is lowered, a lot of work remains to be done to show that the Parisi replica symmetry breaking solution [3] of the SK model is exact or, at least, qualitatively correct. There is, however, a simpler mean-field type model, the Ising spin glass on a Bethe tree [4] for which Chayes, Chayes, Sethna and Thouless have been able to perform a rather complete analysis [5]. Their conclusions which are mathematically rigorous are qualitatively similar to those obtained in the Parisi solution. Heuristic analyses of the short-range Edwards-Anderson spin glass [6] have been carried out in [7–9]. The picture that emerges is still somewhat controversial, but some of the main features of the low-temperature phase of the short-range Ising spin glass on a sufficiently high-dimensional lattice appear to be reminiscent of the Parisi solution [7, 8]. (For example, the space of equilibrium states, at small T and $h=0$, appears to exhibit an ultrametric structure [8].)

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