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The Chandrasekhar Theory of Stellar Collapse as the Limit of Quantum Mechanics

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Dedicated to Walter Thirring on his 60th birthday

Abstract. Starting with a "relativistic" Schrödinger Hamiltonian for neutral gravitating particles, we prove that as the particle number $N \to \infty$ and the gravitation constant $G \to 0$ we obtain the well known semiclassical theory for the ground state of stars. For fermions, the correct limit is to fix $GN^{2/3}$ and the Chandrasekhar formula is obtained. For bosons the correct limit is to fix GN and a Hartree type equation is obtained. In the fermion case we also prove that the semiclassical equation has a unique solution – a fact which had not been established previously.

Historical Remarks and Background

There are two principal elementary models of stellar collapse: neutron stars and white dwarfs. In the former there is only one kind of particle which, since it is electrically neutral, interacts only gravitationally. The typical neutron kinetic energy is high, however, so it must be treated relativistically. Unfortunatly, the mass and density are also large enough that general relativistic effects are important. For white dwarfs, on the other hand, there are two kinds of nonneutral particles: electrons and nuclei. Because the density is not too large, it is a reasonable approximation to ignore general relativistic effects (although these effects might be important for stability considerations [29]); the nuclei (because of their large mass) can be treated nonrelativistically but the electrons must be treated relativistically. The Coulomb interaction is usually accounted for by the simple assumption that local neutrality requires the nuclear charge density to be equal to the electron charge density, in which case the problem reduces to calculating the electron density. (There are, in fact, electrostatic exchange and correlation effects [28, 29], but these are small by a factor $\alpha = 1/137$.)

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