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Localization in General One-Dimensional Random Systems

II. Continuum Schrödinger Operators

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Dedicated to Walter Thirring on his 60th birthday

Abstract. We discuss two ways of extending the recent ideas of localization from discrete Schrödinger operators (Jacobi matrices) to the continuum case. One case allows us to prove localization in the Goldshade, Molchanov, Pastur model for a larger class of functions than previously. The other method studies the model $-\Delta + V$, where V is a random constant in each (hyper-) cube. We extend Wegner's result on the Lipschitz nature of the ids to this model.

1. Introduction

Localization for continuum and discrete random Schrödinger operators has been heavily studied. This note contributes to this literature. Our main goal is to extend to the study of operators on $L^2(R^\nu)$ [especially $L^2(R)$] a set of ideas recently developed to discuss localization for operators on $\ell^2(\mathbb{Z}^\nu)$. These ideas, which have their roots in work of Carmona [2, 3], were developed by Kotani [14, 15] and brought to fruition in Delyon-Levy-Souillard [5–7] and Simon-Wolff [22, 23, 21]. As a by-product, we will extend Wegner's result on the Lipschitz nature of the integrated density of states to certain continuum models.

The models that we will study can be described as follows: Let (Ω, μ) be a probability measure space and let $\{T_x(\omega)\}$ be a one-parameter group of μ-preserving transformations on Ω which is ergodic. Let F be a measurable function from Ω to $\mathbb{R}$. We want to study the family of Schrödinger operators on $L^2(\mathbb{R}^\nu)$:

$$-\Delta + q_\omega(x),$$

where $q_\omega(x) = F(T_x(\omega))$. We always suppose that, for a.e. ω: $q_\omega(x)$ is continuous in x and

$$|q_\omega(x)| \leqq C_\omega(1 + |x|^2)$$

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