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On Global Solutions of the Maxwell-Dirac Equations*

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Dedicated to Walter Thirring on his 60[th] birthday

Abstract. We prove, for the Maxwell-Dirac equations in $1+3$ dimensions, that modified wave operators exist on a domain of small entire test functions of exponential type and that the Cauchy problem, in $R^+ \times R^3$, has a unique solution for each initial condition (at $t=0$) which is in the image of the wave operator. The modification of the wave operator, which eliminates infrared divergences, is given by approximate solutions of the Hamilton-Jacobi equation, for a relativistic electron in an electromagnetic potential. The modified wave operator linearizes the Maxwell-Dirac equations to their linear part.

1. Introduction

Our basic understanding of electromagnetic interactions started with the fundamental discovery made by P. A. M. Dirac of the relativistic electron equation. After years of development it was only in the late forties that Schwinger, Feynman, and Tomonaga formulated Quantum Electro-Dynamics, the theory of fundamental interaction between electrons and photons. This theory, which starts with quantum operator-valued Maxwell-Dirac (M–D) coupled equations, ends up with a set of rules deduced non-mathematically from the previous, permitting to calculate with very high precision different electromagnetic processes. These are the famous Feynman rules. One can take the point of view that all the physics we need is contained in this set of rules, and therefore this is the only thing that matters. However it is quite tempting to believe that, though we did not deduce these rules mathematically, still the quantum M–D equations must have some sense.

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* This work is dedicated to Walter Thirring upon the occasion of his sixtieth birthday with appreciation and friendship