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The Integration of *G*-Invariant Functions and the Geometry of the Faddeev-Popov Procedure

Z. Jaskólski

Institute of Theoretical Physics, University of Wrocław, PL-50-205 Wrocław, ul. Cybulskiego 36, Poland

Abstract. Various versions of the Fubini theorem on the principal fibre bundle are derived. The formal generalizations of these theorems are used as a basic tool for the investigation of the geometrical setting of the Faddeev-Popov procedure in the cases of pure Yang-Mills theories, spinless particle and Polyakov's bosonic string.

1. Introduction

In general the gauge theories can be divided into two classes: those which are invariant under true gauge group transformations and those which are invariant under infinitesimal transformations whose commutators do not necessarily close and involve field-dependent structure constants. There exists a general approach to functional quantization of both types of gauge theories developed along Hamiltonian lines by Fradkin and his collaborators [1] and recently reviewed by Henneaux [2]. This approach is based on a functional integral over paths in the canonical phase space suitably extended by additional ghost degrees of freedom. Due to the so-called Fradkin theorem [1] such an integral is formally equivalent to the phase-space functional integral over physical (independent) degrees of freedom of a constrained system [3]. On the other hand in the case of the first type of gauge theories there is a covariant functional approach introduced by Faddeev and Popov in the case of Yang-Mills theories [4]. One starts with the functional integral over configurations of fields and then by the so-called Faddeev-Popov trick extracts the volume of the gauge group from this integral. The functional integral obtained in this way leads to the perturbative expansion with correct Feynman diagrams [5]. Soon after the discovery of the Gribov ambiguity [6] it was recognised that its origin is closely related to the global geometry of the space of fields [7]. It is known that under some assumptions about an action of the gauge group the space of connections $\mathscr{C}$ has a structure of a principal fibre bundle over the orbit space with the group $\mathscr{G}$ of local gauge transformations as a structure group [8, 9]. In this bundle there exists a connection determined by a natural