

# Level One Representations of the Simple Affine Kac-Moody Algebras in Their Homogeneous Gradations

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**Abstract.** Using the central charge of the Virasoro algebra as a clue, we recall the known constructions of the  $A$ ,  $D$ ,  $E$  algebras and discuss new Bosonic constructions of the non simply laced affine Kac-Moody algebras: the twisted  $A$ ,  $D$ ,  $E$  and the  $B$ ,  $C$ ,  $F$ , and  $G$  algebras. These involve interacting Fermions and a generalization of the Frenkel-Kac sign operators which do not form a 2-cocycle when the horizontal algebra has more than one short simple root.

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