Commun. Math. Phys. 111, 123–135 (1987)

Communications in
Mathematical
Physics

The Universal Structure of Local Algebras

D. Buchholz[1], C. D'Antoni[2,]**, and K. Fredenhagen[1,]*

[1] II. Institut für Theoretische Physik, Universität Hamburg, D-2000 Hamburg 50, Federal Republic of Germany
[2] Dipartimento di Matematica, Università di Roma, La Sapienza, Rome, Italy

Abstract. It is shown that a few physically significant conditions fix the global structure of the local algebras appearing in quantum field theory: it is isomorphic to that of $\mathfrak{R}\bar{\otimes}\mathfrak{Z}$, where $\mathfrak{R}$ is the unique hyperfinite factor of type III_1 and $\mathfrak{Z}$ the center of the respective algebra. The argument is based on results in [1, 2] relating to the type of the local algebras and an improvement of an argument in [3] concerning the "split property."

1. Introduction

Since the very beginnings of the algebraic approach to quantum field theory [4], there has been continuous interest in the structure of the local algebras appearing in this setting. This interest originates from the insight that the entire physical information of a quantum field theory is encoded in the map

$$\mathcal{O}\rightarrow\mathfrak{A}(\mathcal{O}), \tag{1.1}$$

assigning to each bounded region $\mathcal{O}$ of Minkowski space a von Neumann algebra $\mathfrak{A}(\mathcal{O})$ which is generated by the observables (respectively fields) associated with the region in question. So there naturally arises the question of the concrete algebraic properties of the images $\mathfrak{A}(\mathcal{O})$ of this map.

It is by now well known that the local algebras $\mathfrak{A}(\mathcal{O})$ are, in generic cases, of type III_1 according to the classification of Connes (cf. [5]). This fact has been established in several models by explicit calculations, and also by more abstract arguments (cf. [6] for a review). But only recently this result has been derived from conditions which seem to be sufficiently general to cover most theories of physical interest [1]. Besides the standard postulates of quantum field theory the only input needed is the assumption that the theory has a scaling limit. This is expected to be the case in renormalizable field theories with an ultraviolet fixed point, hence in particular in all theories which are asymptotically free.

* Heisenberg fellow
** Research supported by Ministro della Pubblica Istruzione and CNR-GNAFA