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On the Relevance of Some Constructions of Highest Weight Modules Over (Super-) Kac-Moody Algebras (In connection with a paper by Wakimoto)

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Abstract. We correct and improve results of Wakimoto on the (ir)reducibility of his construction of $A_1^{(1)}$ highest weight modules (HWM). For a very large class of (super-) Kac-Moody algebras we argue that such a HWM is most relevant when it is isomorphic to a proper factor-module of the corresponding reducible Verma module with the same highest weight. In the same situation we present a general procedure to check the reducibility of the HWM in consideration.

1.

This note has several purposes. First (in Sect. 2) we correct a statement (Theorem 2) in [1] concerning irreducibility of highest weight modules over $A_1^{(1)}$ constructed there. We also show how to obtain similar theorems for a much larger class of algebras, namely, semisimple Lie algebras, (super-) affine Lie algebras, basic classical Lie super-algebras. For completeness we give the proof of the corrected statement (Theorem A). Next (in Sect. 3) we comment on the relevance of such highest weight module (HWM) constructions in the same general setting. Our point is that unless a physical application is involved a HWM $V(\Lambda)$ with highest weight Λ is of interest when the corresponding Verma module $M(\Lambda)$ is reducible while $V(\Lambda)$ is isomorphic to a proper factor-module of $M(\Lambda)$ and especially when $V(\Lambda)$ is irreducible. We present a general procedure to check the reducibility of $V(\Lambda)$ in the cases when $M(\Lambda)$ is reducible. We apply this procedure to the HWM constructed in [1] and obtain a result (Theorem B) on the reducibility of a large class of these HWM.

2.

In [1] Wakimoto constructed a family of highest weight modules $\pi_{\mu\nu}$ over the affine Lie algebra $A_1^{(1)}$ parametrized by $(\mu, \nu) \in \mathbb{C}^2$. We shall not repeat the construction of the $\pi_{\mu\nu}$ representation spaces $V(\mu, \nu)$ from [1]. We need only the