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An Explicit Construction of a Class of Suspensions and Autonomous Differential Equations for Diffeomorphisms in the Plane

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Abstract. From a large class of diffeomorphisms in the plane, which are known to produce chaotic dynamics, we explicitly construct their continuous suspension on a three dimensional cylinder. This suspension is smooth (C^1) and can be characterized by the choice of two smooth functions on the unit interval, which have to fulfill certain boundary conditions. For the case of entire Cremona transformations, we are able to construct the corresponding autonomous differential equations of the flow explicitly. Thus it is possible to relate properties of discrete maps to those of ordinary differential equations in a quantitative manner. Furthermore, our construction makes it possible to study the exact solutions of chaotic differential-equations directly.

1. Introduction

For the description of dynamical long time behavior of complex systems, two kinds of models have been studied in the literature. The classical models are based on ordinary differential equations, which means that a continuous flow of time is considered. More recently models with discrete time-steps τ have become popular, especially for the description of erratic or chaotic behavior [1]. In this way, the frequencies of the dynamical system above a maximal frequency $\omega_{\max} = \frac{2\pi}{\tau}$ are neglected. These models are based on ordinary difference equations and have been successful in modelling different routes to turbulence [2]. In a general sense it is clear that both of these approaches are equivalent in the description of the different transitions that occur before chaotic behavior sets in. However, little is known about how a given property of a discrete model can be translated into a continuous model and vice versa. For instance it is known for a unimodal map on the interval that the order of its maximum determines its universality class in the period-doubling route to chaos [3]. But the corresponding criterion for differential equations is not clear [4]. The main geometrical argument used in this work is based on the concept of Poincaré-maps [5]. This is a