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String Theory and Loop Space Index Theorems*

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Abstract. We study index theorems for the Dirac-Ramond operator on a compact Riemannian manifold. The existence of a group action on the loop space makes possible the definition of a character valued index which we calculate by using a two-dimensional sigma model with $N=1/2$ supersymmetry. We compute the Euler characteristic, the Hirzebruch signature and the Dirac-Ramond genus of loop space. We compare our results to the calculations made by using the Atiyah-Singer character-valued index theorem.

In this paper we will discuss some attempts at calculating the index of some differential operators acting on infinite dimensional spaces. In particular we will study some of the topological properties of the twisted Dirac-Ramond operator on the loop space $\mathscr{L}(M)$ of a compact Riemannian manifold M. The first problem that one has to face in studying the index on an infinite dimensional manifold is that the kernel of the operator may be infinite dimensional. One therefore needs a device which will split this infinite degeneracy and allow one to work with finite dimensional subspaces of the kernel. A natural way of dealing with this obstacle is to look for a character valued index (G-index) [1–4] which will split the kernel into finite dimensional pieces transforming according to different representations of some group. Witten [5] has emphasized that the equivariant Dirac-Ramond operator provides such an example. The loop space $\mathscr{L}(M)$ has an S^1 action given by sending the loop parameter σ to $\sigma+\Delta$, where Δ is a constant. Representations of S^1 are labelled by an integer n which corresponds to the two-dimensional momentum of the string state. In the Ramond case, the number of states for each value of n is finite.

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