

# Hyperkähler Metrics and Supersymmetry

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**Abstract.** We describe two constructions of hyperkähler manifolds, one based on a Legendre transform, and one on a symplectic quotient. These constructions arose in the context of supersymmetric nonlinear  $\sigma$ -models, but can be described entirely geometrically. In this general setting, we attempt to clarify the relation between supersymmetry and aspects of modern differential geometry, along the way reviewing many basic and well known ideas in the hope of making them accessible to a new audience.

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