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Yang–Lee Zeros of a Planar Ising Model with a Boundary Magnetic Field

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Abstract. A planar Ising ferromagnet is investigated with a magnetic field acting on one surface. The Yang–Lee zeros associated with this field are located exactly on the imaginary axis and their limiting distribution is given. Above the critical temperature, this distribution has a gap, near which the pair correlation for spins in the surface exhibits critical behaviour. The zeros of certain antiferromagnets are located, in particular those for an antiferromagnetic ring coupled ferromagnetically to a planar Ising ferromagnet.

1. Introduction

Since the seminal work of Yang and Lee [1, 2], the location of zeros of partition functions, in particular its limiting behaviour for systems of infinite volume, has been a cornerstone in the statistical-mechanical treatment of phase transitions. For instance, in the Ising ferromagnet with pair interactions and an applied magnetic field h (in units of $k_B T$) acting on all spins, no matter what the integer dimension d is, the zeros all lie on the imaginary h axis [2]. The generally accepted picture is:

1. For $T > T_c(d)$, $T_c(d)$ being the d-dimensional critical temperature, there is a window $(-ih_g, ih_g)$ uniformly free of zeros. As $T \to T_c(d)+$, $h_g \sim (T - T_c(d))^{\Delta}$, where Δ is the usual gap exponent. For $T < T_c(d)$, there is a non-zero density of zeros at $h = 0$ in the infinite volume limit.

2. The density $G(ih, T)$ of zeros has a branch point singularity near h_g of the type

$$G(ih, T) \sim (h - h_g)^{\sigma}, \tag{1}$$

so that the magnetisation, which is essentially the Hilbert transform of G as a function of h, has the same singularity structure. It is periodic, period $2\pi i$ and analytic in the strip $-\pi < \operatorname{Im} h < \pi$ provided it is cut on the imaginary h axis, $\pm (h_g, \pi) \bmod 2\pi$.

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