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Topological Charge of (Lattice) Gauge Fields

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Abstract. Using recently derived explicit formulae for the 2- and 3-cochains in SU(2) gauge theory, we are able to integrate the Chern-Simons density analytically. We arrive – in SU(2) – at a local algebraic expression for the topological charge, which is the sum of local winding numbers associated with the corners (lattice points) of the cells covering the manifold plus contributions from possible isolated gauge singularities which manifest themselves as "vortices" in the 1-, 2- or 3-cochains. Among others we consider hypercubic geometry – i.e. covering the manifold by hypercubes – which is of particular interest to lattice Monte Carlo applications. Finally, we extend our results to SU(3) gauge theory.

I. Introduction

Differentiable SU(N) gauge fields on a compact 4-manifold $\mathbb{M}$ carry a topological charge [1]

$$Q=-\frac{1}{32\pi^2}\int_{\mathbb{M}} d^4x\varepsilon_{\mu\nu\varrho\sigma}\mathrm{Tr}[F_{\mu\nu}F_{\varrho\sigma}], \tag{1}$$

where

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu+[A_\mu, A_\nu]. \tag{2}$$

The charge Q is a measure for topologically non-trivial properties of the gauge fields, which have been argued to play an important role in the physics of the vacuum of QCD and SU(N) gauge theories.

Preliminary results of calculations of Q in SU(2) gauge theory on the lattice [2, 3] hold out hope for a quantitative resolution of the U(1) problem [4]. The recent finding [5] that the vacuum of the quantized (pure) SU(2) gauge theory possesses an underlying instanton structure brings us furthermore in touch with semi-classical ideas of the QCD vacuum [6] and a possible mechanism for chiral symmetry breaking [7], which could be the beginning of a better understanding of the non-perturbative phenomena of these theories.