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Null Vectors, Spinors, and Strings

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Abstract. It is shown how, in the frame of the Cartan-conception of spinors, the old theorems on *minimal surfaces*, as generated from null-curves, formulated by Enneper-Weierstrass (1864–1866) for 3-dimensional ordinary space, and by Eisenhart (1911) for 4-dimensional space-time, may be reformulated in terms of *complex* 2- and 4-component projective spinors respectively. For the corresponding *real* (Majorana) spinors instead the same procedure naturally leads to *strings* in 3-dimensional and 4-dimensional space-time ($\mathbb{R}^{2,1}$ and $\mathbb{R}^{3,1}$). It is suggested that this close connection with Cartan-spinors, and the corresponding (projective) null-geometry, may be the clue for understanding the fundamental nature of strings.

1. Introduction

For more than a century [1], mathematicians have known the fundamental, elementary character of null vectors and null lines and, in particular, of their property to generate minimal surfaces. These played subsequently a central role in several later developments of mathematics, geometry and, especially, of those fundamental branches of physics which are based on classical and quantum field theories.

E. Cartan discovered spinors in 1913 [2] in searching for new representations of rotation groups. However, from his subsequent work [3], it clearly appears how he was especially struck by the equivalence of what he named "simple spinors" with null or isotropic vectors and totally null planes. A two-component spinor, in his words: "est donc en quelque sort un vecteur isotrope orienté ou polarisé" and, in general: "tout spineur simple peut être défini d'une manière concrete comme un v-vecteur isotrope polarisé." This equivalence may now be expressed in modern, perhaps more rigorous, language as a bijective map: "the Cartan-map" [4] between simple or pure-spinor-directions and geometrical elements (totally null planes, quadric Grassmanians or flags) in projective spaces. This Cartan-map is, in