Commun. Math. Phys. 107, 337–356 (1986)

Communications in
Mathematical
Physics

Classical Spin Systems in the Presence of a Wall: Multicomponent Spins

Jürg Fröhlich[1] and Charles-Edouard Pfister[2]

[1] Theoretische Physik, ETH-Hönggerberg, CH-8093 Zürich, Switzerland
[2] Départements de mathématiques et de physique E.P.F.-L., CH-1015 Lausanne, Switzerland

Abstract. In this paper we investigate classical spin systems on a semi-infinite lattice. We establish detailed properties of such systems near the surface layer. For the Ising- and the classical XY models on a semi-infinite lattice we study the phase diagram, the critical properties and the decay of spin-spin correlations near the surface layer.

1. Introduction

1.1. The Models

This paper is devoted to the study of some surface problems for classical bounded spin systems with a continuous internal symmetry group G. We consider models on a semi-infinite sublattice $\mathbb{L}$ of $\mathbb{Z}^3$, say $\mathbb{L}=\{x=(x^1,x^2,x^3)\in\mathbb{Z}^3 : x^3\geqq 0\}$. We propose to study the behaviour of the system near the boundary surface $\sum$, $\sum=\{x\in\mathbb{L}; x^3=0\}$. Let us introduce the simplest model of this kind. The spin at $x\in\mathbb{L}$ is described by a unit vector in $\mathbb{R}^n$, $S(x)=(S^1(x),\ldots,S^n(x))$, $n\geqq 1$, and the Hamiltonian is

$$-\sum_{\{x,y\}} K(x,y)S(x)\cdot S(y)\,, \tag{1.1}$$

where $S(x)\cdot S(y)$ is the Euclidean scalar product in $\mathbb{R}^n$. We consider only short-range interactions and, for the sake of simplicity, we take $K(x,y)=0$ if x and y are *not* nearest neighbours. If x and y are nearest neighbours,

$$K(x,y)=K \quad \text{if} \quad \{x,y\}\not\subset\sum\,, \tag{1.2}$$

and

$$K(x,y)=J \quad \text{if} \quad \{x,y\}\subset\sum\,. \tag{1.3}$$

The inverse temperature β is one, and we investigate the behaviour of the model when J and K are varied, and (primarily) for $n\geqq 2$.