Commun. Math. Phys. 106, 105–135 (1986)

Communications in
Mathematical
Physics

Specifications and Martin Boundaries for $\mathscr{P}(\Phi)_2$-Random Fields

Michael Röckner

Fakultät für Mathematik, Universität Bielefeld, D-4800 Bielefeld 1, Federal Republic of Germany

Abstract. It is shown that $\mathscr{P}(\Phi)_2$-Gibbs states in the sense of Guerra, Rosen and Simon are given by a specification. The construction of the specification is based on finding a proper version of the interaction density given by the polynomial $\mathscr{P}$. The existence of this version follows from the fact that all powers of the solution of a Dirichlet problem for an open bounded set U with boundary data given by a distribution are integrable on U. As a consequence the Martin boundary theory for specifications can be applied to $\mathscr{P}(\Phi)_2$-random fields. It follows that any $\mathscr{P}(\Phi)_2$-Gibbs state can be represented in terms of extreme Gibbs states. In certain cases the extreme Gibbs states are characterized in terms of harmonic functions. It follows, in particular, that for any given boundary condition introduced so far the associated cutoff $\mathscr{P}(\Phi)_2$-measure has a representation as an integral over harmonic functions.

Contents

1. Introduction

The aim of this paper is to construct "specifications" (cf. [F, P1]) for $\mathscr{P}(\Phi)_2$-random fields. These are known as basic non-trivial continuous models in Euclidean quantum field theory. Here $\mathscr{P}$ is a semibounded polynomial of one variable, Φ is the "field" and the index indicates 2 dimensions (cf. below and [S, Gl/J] for the precise definition). These models, which are usually realized as probability measures on a