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Global Existence of Time-Dependent Yang-Mills-Higgs Monopoles

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Abstract. We study the Cauchy problem for non-abelian Yang-Mills-Higgs theory in $(3+1)$-dimensional Minkowski spacetime. With suitable conditions on the background fields and a suitable choice of a Sobolev space for the subtracted gauge potentials and the Higgs field, we establish local existence. We then prove global existence by showing that an appropriate norm of the solutions cannot blow up in a finite time.

I. Introduction

Irving Segal [1], in 1963, introduced a general existence theory for semi-linear evolution equations. In 1979 Segal [2] himself showed that classical Yang-Mills theory could be cast into a suitable form to make use of this general theory and he showed that the local-in-time Cauchy problem could be solved. This means that if one is given regular initial data on $\mathbb{R}^3$ at some initial time (call it $t=0$), there exists a unique smooth solution to the field equations compatible with the initial data over some finite time interval $(-t_0, t_0)$.

This result was improved on in 1982 by Ginibre and Velo [3] who added a Higgs field to the Yang-Mills potential and showed that local-in-time existence still held. The next major step forward was achieved, also in 1982, by Eardley and Moncrief [4] who independently derived the local existence result for Yang-Mills-Higgs theory and then extended this to obtain a global existence proof, that is to show that the time existence interval $(-t_0, t_0)$ can be made unboundedly large.

The major interest in classical solutions to the Yang-Mills-Higgs equations is due to the existence of magnetic monopole solutions (which demand that the Yang-Mills potential $\mathbf{A}$ falls off like $1/r$) and non-trivial topologies in the Higgs field (which require that the Higgs field remain finite at infinity). However, all the existence results to date demand that both the Higgs field and the Yang-Mills potential be square-integrable on $\mathbb{R}^3$, which naively demands that everything fall off faster than $r^{-3/2}$, and are incompatible with the magnetic monopoles and with non-trivial topologies.