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# Existence of Localized Solutions for a Classical Nonlinear Dirac Field

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**Abstract.** We prove the existence of stationary states for nonlinear Dirac equations of the form:

$$i\gamma^\mu \partial_\mu \psi - m\psi + F(\bar{\psi}\psi)\psi = 0.$$

We seek solutions which are separable in spherical coordinates and we use a shooting method for solving the associated problem of ordinary differential equations.

## 1. Introduction

In this paper we prove the existence of stationary states for nonlinear Dirac equations of the form

$$i \sum_{\mu=0}^{3} \gamma^\mu \partial_\mu \psi - m\psi + F(\bar{\psi}\psi)\psi = 0 \tag{1.1}$$

under certain hypothesis on $F$.

The notation is the following: $\psi$ is defined on $\mathbb{R}^4$ with values in $\mathbb{C}^4$, $\partial_\mu = \partial/\partial x_\mu$, $m$ is a positive constant, $\gamma^\mu$ are $4 \times 4$ matrices given by

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^k = \begin{pmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{pmatrix} \quad k = 1, 2, 3, \text{ where}$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

and $\bar{\psi}\psi = (\gamma^0\psi, \psi)$, where $(,)$ is the usual scalar product in $\mathbb{C}^4$.

Nonlinear spinor fields giving rise to equations of the form (1.1) have been considered first by D. Ivanenko [7], H. Weyl [22], and by W. Heisenberg [6] in his unified theory of elementary particles. Later R. Finkelstein, C. F. Fronsdal and P. Kaus [4] considered the case of a spinor field with several types of fourth order self couplings. But it was M. Soler [16] who was the first to investigate the stationary