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Truncated Gamow Functions, α-Decay and the Exponential Law

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Abstract. For a quantum mechanical two-body s-wave resonance we prove that the evolution of square integrable approximations of the Gamow function is outgoing and exponentially damped. An error estimate is given in terms of resonance energy and width and the time variable. Furthermore the energy distribution is given explicitly. We obtain the Breit-Wigner form. The results are used in an α-decay model to prove general validity of the exponential decay law for periods of several lifetimes.

1. Introduction

The success of quantum mechanics to describe α-decay from a heavy nucleus is well-known. It was established by Gamow [3] and Gurney and Condon [4]. To simplify the problem the discussion is conveniently based on the following one-dimensional model: An α-particle is considered in a spherically symmetric potential $V = V(r)$ comprising a short-range negative piece from the attraction between nucleons and a positive piece of longer range from the Coulomb repulsion between protons. The large barrier thus defined by V has the effect that it confines the α-particle for a long period until it eventually escapes by tunneling. The relatively small energy differences of α-particles escaping from RaA, RaC', and Ur (examples taken from [3, 4]) account in this model for the extremely large decay rate differences observed.

Although the decay rate formulas obtained in [3, 4] are identical, the derivations are different. The idea in [3], that the α-particle is associated with a complex energy $E - i\Gamma/2$ $(= k_0^2,\ k_0 = \alpha - i\beta)$, an exponentially increasing purely outgoing ($\simeq e^{ik_0 r}$ for r large) space function $f(k_0, r)$ and consequently (?) an outgoing exponentially damped state, is missing in [4] and, it seems, in modern textbook derivations. E and Γ^{-1} are the energy and the lifetime, respectively.

The purpose of this paper is to put the above idea on a rigorous footing, first of all to prove that the evolution of some square integrable approximations of $f(k_0, r)$ is in fact outgoing and exponentially damped. To do this we most conveniently