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The Classical Limit of the Relativistic Vlasov–Maxwell System

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Abstract. Solutions of the relativistic Vlasov–Maxwell system of partial differential equations are considered in three space dimensions. The speed of light, c, appears as a parameter in this system. For smooth Cauchy data, classical solutions are shown to exist on a time interval that is independent of c. Then, using an integral representation for the electric and magnetic fields due to Glassey and Strauss [6], conditions are given under which solutions of the relativistic Vlasov–Maxwell system converge in a pointwise sense to solutions of the non-relativistic Vlasov–Poisson system at the asymptotic rate of $1/c$, as c tends to infinity.

Introduction

Consider the Cauchy problem

$$\text{(RVM)}\qquad \begin{cases} \partial_t f + \hat{v}\cdot\nabla_x f + (E + c^{-1}\hat{v}\times B)\cdot\nabla_v f = 0, \\ \partial_t E = c\nabla\times B - 4\pi j \qquad \nabla\cdot E = 4\pi\rho, \\ \partial_t B = -c\nabla\times E \qquad \nabla\cdot B = 0, \\ \rho(x,t) = \int f(x,v,t)\,dv \qquad j(x,t) = \int f(x,v,t)\hat{v}\,dv, \end{cases}$$

where

$$\hat{v} = (1 + c^{-2}v^2)^{-1/2}v,$$

and the initial conditions are

$$\text{(RIC)}\qquad \begin{cases} f(x,v,0) = f_0(x,v), \\ E(x,0) = E_0(x), \\ B(x,0) = B_0(x). \end{cases}$$

Here x and v are points in $\mathbb{R}^3$ representing position and momentum respectively. f gives the number density of a collisionless plasma consisting of a single species of charged particle acting under its selfinduced Lorentz force, $E + c^{-1}\hat{v}\times B$. The parameter c is the speed of light. (RVM) is a relativistic version of the classical Vlasov–Maxwell system (VM), which may be obtained from (RVM) by replacing $\hat{v}$ with v. The formulation of the relativistic version is discussed in [11 and 13].