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Critical Scaling for Monodromy Fields

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Abstract. The large scale asymptotics of the correlations for a family of two dimensional lattice field theories is calculated at the critical "temperature".

Introduction

This paper is devoted to an examination of the large scale asymptotics of the correlation functions for monodromy fields at the critical point. Monodromy fields on the lattice were introduced in [25] based on a natural generalization of the two dimensional Ising field. They are lattice versions of the continuum fields studied by Sato, Miwa and Jimbo in [33] and have also appeared in work on the Federbush and massless Thirring model again in a continuum setting [29, 30, 32]. In [25] the asymptotics of the correlations were examined in the limit that sends the lattice spacing to zero and the "temperature" to the critical point so that the correlation length remains fixed, (massive scaling regime). In this paper we examine the large scale asymptotics of the correlations at the critical point (massless regime) [18]. The original inspiration for this work was the calculation of the critical asymptotics for the two dimensional Ising model. The Ising field is a special case of a monodromy field in the following sense. For each $p \times p$ matrix M we define a field operator $\sigma_m(M)$ ($m \in \mathbb{Z}^2$). The terminology "monodromy field" used in connection with $\sigma_m(M)$ is motivated by the fact that it is possible to "create" monodromy (M) located at $m \in \mathbb{Z}^2$ in the solution to a certain linear difference equation on the lattice through a formula involving $\sigma_m(M)$ (see 4.1 in [25]). When M is the scalar -1 (1×1 matrix) one has the Ising model in the sense that the vacuum expectation of a product $\sigma_{m_1}(-1) \cdots \sigma_{m_n}(-1)$ gives the *square* of an Ising correlation (see [20] and [21]).

The critical scaling limit of the Ising model is of interest for a variety of reasons related to the renormalization group analysis of critical phenomena [18]. Attached to a critical point in a statistical mechanical system are various critical exponents which measure how different physical quantities behave as the critical temperature is approached [13]. It is observed experimentally that wide varieties of physical systems have the same critical exponents [13]. To "explain" this universality one posits a connection between the critical exponents which measure the behaviour of the system near the critical point and the large scale asymptotics of