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Spontaneous Symmetry Breakdown in the Abelian Higgs Model

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Abstract. For the abelian Higgs model we introduce a new gauge invariant observable which in Landau gauge is $\phi(x)\,\bar{\phi}(y)$. In three or more dimensions this observable is used to show that the global gauge symmetry is spontaneously broken in the lattice theory for a suitable range of parameters. This observable also provides a gauge invariant order parameter for the phase transition in this model.

1. Introduction

The Higgs mechanism plays an important role in determining the particle structure of gauge field theories. This mechanism provides a natural way for the gauge bosons to acquire a mass. The standard explanation of this mass generation assumes that the global gauge symmetry is spontaneously broken [2].

One of the simplest gauge theories which is believed to exhibit spontaneous symmetry breaking and the Higgs mechanism is the abelian Higgs model. It has been rigorously established in Euclidean lattice versions of this model that there exists a phase in which the photon is massless and a phase in which it is massive [1, 7, 8, 17, 18].

These proofs of the Higgs mechanism are quite different from the heuristic explanation and do not address the question of spontaneous symmetry breaking. In this paper we introduce a new order parameter and use it to prove spontaneous symmetry breaking in the abelian Higgs model in dimension three or more. Our result is proven for the Euclidean lattice version of the model with a noncompact action.

In the abelian Higgs model spontaneous symmetry breakdown would mean that $\langle\phi(x)\rangle = \phi_0 \neq 0$, where $\phi(x)$ is the Higgs field. This would mean that the Higgs field has long range order (LRO) like the spins in a ferromagnet at low temperature. However, $\phi(x)$ is not a gauge invariant observable, so this picture

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