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Integrable Non-Linear σ Models with Fermions

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Abstract. The two-dimensional non-linear σ model on a Riemannian symmetric space $M = G/H$ is coupled to fermions with quartic self-interactions. The resulting hybrid model is presented in a gauge-dependent formulation, with a bosonic field taking values in G and a fermionic field transforming under a given representation of the gauge group H. General criteria for classical integrability are presented: they essentially fix the Lagrangian of the model but leave the fermion representation completely arbitrary. It is shown that by a special choice for the fermion representation (derived from the adjoint representation of G by an appropriate reduction) one arrives naturally at the supersymmetric non-linear σ model on $M = G/H$. The issue of quantum integrability is also discussed, though with less stringent results.

1. Introduction and Summary

Generalized non-linear σ models, also called chiral models, are prime examples of field theories with non-trivial dynamical content which have a geometric origin, and it is well known that they are in many respects closely related to non-abelian gauge theories. (For some of the many aspects of this relation, see for example [1–4].) One of the most attractive features of these models is that classically they provide examples of integrable systems in two-dimensional space-time. Namely, it is known that the non-linear equations of motion are precisely the compatibility conditions for a certain linear system of first-order partial differential equations (Lax pair) containing a spectral parameter, and that this hidden symmetry gives rise to infinite series of local as well as non-local conservation laws [5–7], whenever the field takes values in a Riemannian symmetric space $M = G/H$ [8–10]. (For reviews, see for example [11, 12].) In the quantum theory, these integrability

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