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The $U(1)$ Higgs Model

II. The Infinite Volume Limit

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Abstract. We construct the infinite volume limit of the $U(1)$ Higgs model in two and three dimensions, and verify some of the Osterwalder-Schrader axioms. The proof uses a combination of renormalization group techniques and correlation inequalities.

1. Introduction

In a previous paper [K 1], we considered the lattice $U(1)$ Higgs model in a finite volume with periodic boundary conditions in dimensions $d=2, 3$. We proved the convergence of this model to a finite limit as the lattice spacing approaches zero. The method of proof was based on the renormalization transformations introduced in [Ba 1–4]. In this paper, we continue our analysis of the model by constructing the infinite volume theory and verifying some of the Osterwalder-Schrader axioms.

This model was previously constructed in two dimensions in the papers [BFS 1–3]. In particular, a collection of useful correlation inequalities was established in [BFS 1]. We use these inequalities to define the infinite volume limit of the continuum theory derived in [K 1]. The limit is shown to satisfy reflection positivity, translation invariance, and a suitable analyticity property.

Since we approach the continuum limit through a sequence of models defined on lattices with a fixed orientation, it is not obvious that the continuum theory satisfies rotation invariance. We prove a result which, when combined with a construction of the infinite volume limit for periodic boundary conditions, implies the rotation invariance of that limit. (Although the correlation inequalities of [BSF 1] fail for periodic boundary conditions, other methods such as the cluster expansion could be used.) Specifically, we prove that two lattices whose orientations differ by a special angle θ_0 (incommensurate with 2π) give the same continuum theory in a finite volume.

The paper is organized as follows. Section 2 contains a statement of the results proved in this paper. In Sect. 3, we construct the infinite volume limit and verify the properties mentioned before. Section 4 contains the results on rotation invariance. Finally, in Sect. 5 we extend the ultra-violet stability results of [Ba 1–4] to include the different boundary conditions used in this paper.

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