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Navier-Stokes Equations for Compressible Fluids: Global Existence and Qualitative Properties of the Solutions in the General Case

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Abstract. We consider the equations which describe the motion of a viscous compressible fluid, taking into consideration the case of inflow and/or outflow through the boundary. By means of some a priori estimates we prove the existence of a global (in time) solution. Moreover, as a consequence of a stability result, we show that there exist a periodic solution and a stationary solution.

1. Introduction

In this paper the motion of a viscous compressible fluid is considered. The motion in a bounded domain $\Omega \subset \mathbb{R}^3$ is described by the following equations

$$\begin{gathered}
\varrho[u_t + u\cdot\nabla u - b] = -\nabla p - Au \quad \text{in} \quad Q_T,\\
\varrho_t + u\cdot\nabla\varrho + \varrho\,\mathrm{div}\,u = 0 \quad \text{in} \quad Q_T,\\
\varrho c_v[\theta_t + u\cdot\nabla\theta] + \theta p_\theta\,\mathrm{div}\,u\\
= \varrho r + \chi\Delta\theta + \frac{\mu}{2}\sum_{i,j}(D_i u^j + D_j u^i)^2 + (\nu-\mu)(\mathrm{div}\,u)^2 \quad \text{in} \quad Q_T,\\
u_{|t=0} = u_0 \quad \text{in} \quad \Omega,\\
u_{|\partial\Omega} = \bar{u}_{|\partial\Omega} \quad \text{on} \quad \Sigma_T,\\
\theta_{|t=0} = \theta_0 \quad \text{in} \quad \Omega,\\
\theta_{|\partial\Omega} = \bar{\theta}_{|\partial\Omega} \quad \text{on} \quad \Sigma_T,\\
\varrho_{|t=0} = \varrho_0 \quad \text{in} \quad \Omega,
\end{gathered} \tag{1.1}$$

where $-A \equiv \mu\Delta + \nu\nabla\,\mathrm{div}$. (See, for instance, Serrin [23].)

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