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Internal Lifschitz Singularities of Disordered Finite-Difference Schrödinger Operators

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Abstract. The integrated density of states has $\mathbf{C}^\infty$-like singularities, $\ln|k(E)-k(E_c)| = -|E-E_c|^{-\nu/2}\varphi_c(E)$, with $\varphi_c > 0$, a milder function at the edges of the spectral gaps which appear when the distribution function of the potential $d\mu$ has a sufficiently large gap. The behaviour of φ_c near E_c is determined by the local continuity properties of $d\mu$ near the relevant edge: $\varphi_c(E) = \mathcal{O}(1)$ if $d\mu$ has an atom and $\varphi_c = \mathcal{O}(\ln|E-E_c|)$ if μ is (absolutely) continuous and power bounded.

Introduction

Let $H_\omega = T + V_\omega$ be a tight-binding Schrödinger operator with disordered potential on $\mathbb{Z}^\nu$ (or an infinite sublattice of it):

$$(H_\omega f)(n) = \sum_m I(n-m)f(m) + V_\omega(n)f(n). \tag{1.1}$$

Here I has compact support and $V_\omega(n)$ are independent, identically distributed (iid) random variables. The (compact) support of their common distribution function $d\mu$ contains at least two points.

Such operators appear in many models for electrons in disordered systems either as finite difference approximations of Schrödinger operators or as restrictions of such operators to subspaces spanned by localized (atomic or Wannier) basis sets.

Let $\mathcal{N}(E, A)$ be the number of eigenvalues of the operator A which are less than E. The integrated density of states (IDS) for H may be defined by

$$k(E) = \lim_{\Lambda \uparrow \mathbb{Z}^\nu} |\Lambda|^{-1} \mathcal{N}(E, H_\omega^\Lambda), \tag{1.2}$$

where H_ω^Λ is a restriction of H to the compact $\Lambda \subset \mathbb{Z}^\nu$ and $|\Lambda|$ is the number of points in Λ.[1]

1 For Schrödinger operators on $\mathbb{R}^\nu$, $|\Lambda|$ is the Lebesgue measure of Λ