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Periodic Solutions in a Model of Pulsar Rotation

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Abstract. A rotating rigid body with ellipsoidal cavity filled with magnetic fluid is considered as a pulsar model. Dynamical equations for the pulsar model are derived and investigated, certain integrable cases are indicated. Three-parameter sets of periodic solutions integrable in terms of elliptic functions of the time variable are obtained. A formula is derived for the period of rotation and magneto-rotational oscillations of the pulsar.

Introduction

It is now generally acknowledged [1] that pulsars (neutron stars) have a solid envelope and a liquid core which has high conductivity (the liquid is plasma) and strong frozen magnetic fields; the liquid core contains the dominating part of the pulsar mass. "Starquakes" happen periodically in pulsars, and they can be observed as glitches of the pulsar rotation period. Asynchronous rotations of the pulsar core and envelope take place during the time intervals between two such phenomena. The relaxation time for the Vela pulsar (PSR 0833-45) is $\tau \approx 6$ years, while its rotation period is $P = 0.089$ s. Therefore the viscosity effects are negligible and an appropriate model of the pulsar core is that of the ideal incompressible magnetic fluid.

The model of the pulsar rotation which is proposed in the present work takes into account magnetic properties of the core and asynchronous rotation of the core and envelope (Sect. 1). The model is applicable for a finite time interval t, $P \ll t < \tau$, between two subsequent starquakes, where the energy losses to the viscous friction and electromagnetic radiation may be neglected.

The dynamical system describing the rotation of the pulsar model is derived in Sect. 2. It is a system of nine ordinary differential equations which are represented in a simple vector form, Eq. (2.12). This system has four first integrals J_k; J_1 is the total energy of the pulsar, J_2 is the total angular momentum squared, J_3 defines the magnitude of the frozen magnetic field, and J_4 is the scalar product of the curl of the fluid velocity by the magnetic intensity vector (Sect. 3).