Commun. Math. Phys. 102, 105–122 (1985)

Communications in Mathematical Physics

B.R.S. Algebras.
Analysis of the Consistency Equations in Gauge Theory

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Abstract. We compute all possible anomalous terms in quantum gauge theory in the natural class of polynomials of differential forms. By using the appropriate cohomological and algebraic methods, we do it for all dimensions of spacetime and all structure groups with reductive Lie algebras.

1. Introduction

It is known that anomalous terms in quantum gauge theory (e.g. chiral anomalies, Schwinger terms) verify consistency equations [1, 2] of cohomological nature [3]. The cohomology of interest is the local cohomology of the Becchi-Rouet-Stora (B.R.S) operator, [4].

An anomalous term Δ verifies a consistency equation: $\delta\Delta = 0$. Δ, (the anomalous term), is the integral of a polynomial in the fields and their derivatives. However, solutions of the form $\Delta = \delta\Delta'$, where Δ' is a similar local expression, are considered as trivial; indeed, in the case of chiral anomalies or Schwinger terms, such trivial solutions may be cancelled by finite renormalization or by redefinition of the local currents respectively.

Setting $\Delta = \int Q$ leads, for Q, to the Eq. [5]

$$\delta Q = dQ' \qquad (*)$$

for some Q'; d is the exterior differential on space-time. We shall say that Q is a *δ-cocycle modulo d*. If Δ is a trivial solution $\Delta = \delta\Delta'$, then Q reads

$$Q = \delta L' + dL \qquad (**)$$

for some L' and L; we say that such a Q is a *δ-coboundary modulo d*. As pointed out before, we are interested in solutions of (*) modulo solutions of the form (**), i.e. in the *δ-cohomology modulo d*.

More precisely, d and δ act as antiderivations on polynomial functions of the gauge potential 1-form A, the ghost field χ and their derivatives with values in differential forms on space-time M. d is the exterior differential on space-time and