Commun. Math. Phys. 102, 89–103 (1985)

Communications in Mathematical Physics

Phase Diagram of the Two-Dimensional Ising Antiferromagnet (Computer-Assisted Proof)

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Abstract. We study the phase diagram of the Ising antiferromagnet on a square lattice in a neighbourhood of ground state critical points $h = \pm 4$, $T = 0$. It leads to a question about the value a_C of the critical activity of the hard-square lattice gas. Using a constructive criterion of uniqueness we prove that $a_C > 1$ and that the phase diagram of the antiferromagnet does not bulge near mentioned critical points. It is a specific feature of this work that the proof was completed with the help of a computer.

1. Introduction

Let us consider the Ising antiferromagnet on a square lattice with spins $\sigma_t = \pm 1$ attached to lattice sites $t \in \mathbb{Z}^2$ and with the Hamiltonian

$$H = \sum_{\langle st \rangle} \sigma_s \sigma_t - h \sum_t \sigma_t, \tag{1.1}$$

where the first sum is over all nearest neighbours $s, t \in \mathbb{Z}^2$, $|s-t| = 1$.

For $|h| < 2d = 4$, there are two different ground configurations $\sigma^{(1)}, \sigma^{(2)}$ in this model:

$$\sigma_t^{(1)} = \begin{cases} +1 & \text{whenever} \quad t_1 + t_2 \quad \text{is even}, \\ -1 & \text{whenever} \quad t_1 + t_2 \quad \text{is odd}, \end{cases}$$

$$\sigma_t^{(2)} = -\sigma_t^{(1)}.$$

These configurations are stable, i.e. they generate extremal Gibbs states $\langle \; \rangle_\beta^{(1)}, \langle \; \rangle_\beta^{(2)}$, corresponding to the Hamiltonian (1.1) and to an inverse temperature $\beta = T^{-1}$, which are for large β only small perturbations of corresponding non-random states concentrated on the configurations $\sigma^{(1)}$, $\sigma^{(2)}$, respectively. It may be shown that these two Gibbs states differ in a region

$$\beta(4 - |h|) > \bar{\mu} \tag{1.2}$$