Commun. Math. Phys. 101, 97–127 (1985)

Communications in Mathematical Physics

Asymptotic Stability of Traveling Wave Solutions of Systems for One-dimensional Gas Motion

Shuichi Kawashima[1] and Akitaka Matsumura[2]

1 Department of Mathematics, Nara Women's University, Nara 630, Japan

2 Department of Applied Mathematics and Physics, Kyoto University, Kyoto 606, Japan

Abstract. The asymptotic stability of traveling wave solutions with shock profile is investigated for several systems in gas dynamics. 1) The solution of a scalar conservation law with viscosity approaches the traveling wave solution at the rate $t^{-\gamma}$ (for some $\gamma > 0$) as $t \to \infty$, provided that the initial disturbance is small and of integral zero, and in addition decays at an algebraic rate for $|x| \to \infty$. 2) The traveling wave solution with Nishida and Smoller's condition of the system of a viscous heat-conductive ideal gas is asymptotically stable, provided the initial disturbance is small and of integral zero. 3) The traveling wave solution with weak shock profile of the Broadwell model system of the Boltzmann equation is asymptotically stable, provided the initial disturbance is small and its hydrodynamical moments are of integral zero. Each proof is given by applying an elementary energy method to the integrated system of the conservation form of the original one. The property of integral zero of the initial disturbance plays a crucial role in this procedure.

Contents