Commun. Math. Phys. 101, 21–46 (1985)

Communications in Mathematical Physics

Constructive Proof of Localization in the Anderson Tight Binding Model

J. Fröhlich[1]*, F. Martinelli[2], E. Scoppola[3], and T. Spencer[1]†

1 The Institute for Advanced Study, Princeton, NJ 08540, USA
2 Dipartimento di Matematica, Università di Roma "La Sapienza", Piazzale A. Moro, 2, I-00185 Rome, Italy
3 Dipartimento di Fisica, Università di Roma "La Sapienza", Piazzale A. Moro, 2, I-00185 Rome, Italy

Abstract. We prove that, for large disorder or near the band tails, the spectrum of the Anderson tight binding Hamiltonian with diagonal disorder consists exclusively of discrete eigenvalues. The corresponding eigenfunctions are exponentially well localized. These results hold in arbitrary dimension and with probability one. In one dimension, we recover the result that all states are localized for arbitrary energies and arbitrarily small disorder. Our techniques extend to other physical systems which exhibit localization phenomena, such as infinite systems of coupled harmonic oscillators, or random Schrödinger operators in the continuum.

1. Introduction and Outline of Paper

In this paper we analyze the spectral properties of Anderson's tight binding Hamiltonian, H, with diagonal disorder, [1]. This operator describes the dynamics of a quantum mechanical particle moving under the influence of a random potential, v. For convenience, we study the discrete case, where the particle may hop on a lattice $\mathbb{Z}^\nu$, but our techniques can be extended to continuous systems. Our main result asserts completeness of the point spectrum and exponential decay of eigenfunctions of H in the band tails, or throughout the spectrum of H provided the disorder is large. This result holds with probability one and in arbitrary dimension ν. Related results have recently been announced by Ya. Gol'dsheid, but his proofs do not seem to have appeared, yet. In one dimension we may combine our techniques

* Work supported in part by National Science Foundations grant MCS-8108814 (A03).
Permanent address: Theoretical Physics, ETH-Hönggerberg, CH-8093 Zürich, Switzerland
† Work supported in part by National Science Foundation grant DMR 81-00417.
Permanent address: Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York, NY 10012, USA