Commun. Math. Phys. 101, 1–19 (1985)

Communications in
Mathematical
Physics

Harmonic Analysis on SL(2, R) and Smoothness of the Density of States in the One-Dimensional Anderson Model

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Abstract. We consider infinite Jacobi matrices with ones off-diagonal, and independent identically distributed random variables with distribution $F(v)dv$ on-diagonal. If F has compact support and lies in some Sobolev space L^1_α, then we prove that the integrated density of states, $k(E)$, is C^∞ in E.

1. Introduction

In this paper, we will study the one-dimensional Anderson model

$$(h_\omega u)(n) = u(n+1) + u(n-1) + V_\omega(n)u(n)$$

on $l^2(Z)$, where $V_\omega(n)$ are independent identically distributed random variables with distribution $d\eta(v)$. The operator restricted to $l^2([0, l-1])$ with $u(-1) = u(l) = 0$ boundary condition is denoted by h^l_ω. This $l \times l$ matrix has eigenvalues $e^l_\omega(1) < \cdots < e^l_\omega(l)$. The integrated density of states, $k(E)$, is defined by

$$k(E) = \lim_{l\to\infty} l^{-1}\#(j \mid e^l_\omega(j) < E).$$

It is a basic result [3, 2, 11], essentially a consequence of the ergodic theorem, that for a.e. ω the limit exists for all E.

It is a result of Pastur [15] that $k(E)$ continuous in E, Craig–Simon [6] show that k is Log–Hölder continuous, i.e. $|k(E) - k(E')| \leqq c_R\{\ln(|E - E'|\}^{-1}$ if $|E| \leqq R$, $|E - E'| < \frac{1}{2}$, and LePage [12] that $k(E)$ is Hölder continuous of some order $\alpha > 0$ in this situation. (The results of [6, 15] hold in great generality.) Here we want to consider greater regularity in E. Without restrictions on $d\eta$, one cannot expect too much more regularity. There is an argument of Halperin [24], essentially

* Research partially supported by USNSF under Grant MCS-81-20833

** Research partially supported by USNSF under Grant MCS-82-01766A01