Commun. Math. Phys. 100, 463–470 (1985)

Communications in
Mathematical
Physics

Anderson Localization for Multi-Dimensional Systems at Large Disorder or Large Energy

Francois Delyon, Yves Lévy, and Bernard Souillard
Centre de Physique Théorique*, Ecole Polytechnique, F-91128 Palaiseau Cedex, France

Abstract. We prove that discrete Schrödinger operators on $\mathbb{Z}^d$ with a random-potential have almost-surely only pure point spectrum and exponentially decaying eigenfunctions for large disorder or large energy. This is the first proof of localization for multi-dimensional Anderson models.

Introduction

Disordered systems are presently widely studied from the mathematical point of view. One of the challenging questions concerns the Anderson localization theory, which in mathematical terms amounts to studying the nature of the spectrum of random self-adjoint operators, such as a discrete Schrödinger equation with a random potential, which is among condensed matter physicists the most popular model for describing the electron propagation in a disordered system. A brief survey of these problems can be found in ref. [12], whereas [11] presents a very large bibliography on them.

In this paper we consider the multi-dimensional discrete Schrödinger operator with a random potential given for $\Psi \in \mathbb{R}^{\mathbb{Z}^d}, d \geqq 1$, by

$$(H^V \Psi)(x) = - \sum_{|y-x|=1} \Psi(y) + V(x)\Psi(x),$$

where $V = \{V(x)\}_{x \in \mathbb{Z}^d}$ is a random potential chosen with a probability distribution μ in $\Omega = \mathbb{R}^{\mathbb{Z}^d}$. For all potentials, H is a self-adjoint operator on $l^2(\mathbb{Z}^d)$ and admits as a core the set of those Ψ with a finite support.

For $d = 1$, it is known that exponential localization occurs even for arbitrarily weak disorder. Several works have dealt with these one-dimensional problems; for the simplest proof of localization for one and quasi one-dimensional systems, and for a set of older references, we refer to a companion paper [3] which uses the same techniques as the present paper.

* Groupe de recherche 048 du CNRS