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The Low-Temperature Behavior of Disordered Magnets

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Abstract. We map out the low-temperature phase diagrams of dilute Ising ferromagnets and predominantly ferromagnetic ferrites, obtaining nonperturbative and essentially optimal conditions on the density of ferromagnetic couplings required to maintain long-range order. We also study mappings of dilute antiferromagnets in a uniform field onto random field ferromagnets.

For the randomly dilute systems, we prove that ferromagnetically ordered states exist at low temperature if the density of ferromagnetic couplings exceeds the (appropriately defined) percolation threshold, thereby extending the result of Georgii to three or more dimensions. We also show that, for these systems, as the temperature tends to zero, the magnetization approaches the percolation probability of the corresponding Bernoulli system. In two dimensions, we prove that low-temperature ordering persists in the presence of antiferromagnetic impurities if the ferromagnetic couplings percolate and if the density of antiferromagnetic couplings is bounded above by the order of the inverse square of the corresponding percolation correlation length. For these systems, we rigorously compute the first order decrease in the zero-temperature nominal spontaneous magnetization, in terms of derivatives of the percolation probability, thereby establishing the existence of ferrimagnetically ordered states. Finally, we introduce a model of a random ferrite which exhibits spontaneous magnetization anticorrelated with the boundary conditions.

1. Organization and Main Results

In this paper we study the phase diagrams and order parameters of disordered Ising magnets. Our main rigorous results concern dilute, predominantly fer-

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