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The Ground State Energy of a Bose Gas with Coulomb Interaction*

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Abstract. Let H_N be the $2N$ particle Hamiltonian

$$H_N = \sum_{i=1}^{2N} (-\Delta_i) + \sum_{i<j=1}^{N} |x_i - x_j|^{-1} + \sum_{i<j=1}^{N} |x_{i+N} - x_{j+N}|^{-1} - \sum_{i,j=1}^{N} |x_i - x_{j+N}|^{-1},$$

where Δ_i is the Laplacian in the variable $x_i \in \mathbb{R}^3$, $1 \leqq i \leqq 2N$. The operator H_N is assumed to act on wave functions $\psi(x_1, \ldots, x_N; x_{N+1}, \ldots, x_{2N})$ which are symmetric in the variables $(x_1, \ldots, x_N)$ and $(x_{N+1}, \ldots, x_{2N})$. Suppose ψ is supported in a set Λ^{2N}, where Λ is a cube in $\mathbb{R}^3$. It is shown that if a normalized wave function ψ can be written as a product of two wave functions

$$\psi(x_1, \ldots, x_N; x_{N+1}, \ldots, x_{2N}) = \psi_1(x_1, \ldots, x_N)\psi_2(x_{N+1}, \ldots, x_{2N}),$$

and the density of particles in Λ is constant, then $\langle \psi | H_N | \psi \rangle \geqq -CN^{7/5}$ for some universal constant C.

1. Introduction

In this paper we study the ground state energy of a Bose gas consisting of equal numbers of positive and negative particles interacting via a Coulomb potential. Thus, if the gas contains $2N$ particles with the N negative particles being located at positions $x_1, \ldots, x_N \in \mathbb{R}^3$, and the positive particles at $x_{N+1}, \ldots, x_{2N} \in \mathbb{R}^3$, its Hamiltonian H_N is given by

$$H_N = \sum_{i=1}^{2N} (-\Delta_i) + \sum_{i<j=1}^{N} |x_i - x_j|^{-1} + \sum_{i<j=1}^{N} |x_{i+N} - x_{j+N}|^{-1} - \sum_{i,j=1}^{N} |x_i - x_{j+N}|^{-1}. \tag{1.1}$$

Here Δ_i denotes the Laplacian in the variable x_i, $1 \leqq i \leqq 2N$.

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