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Existence of Stark Ladder Resonances

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Abstract. We show that resonances, in the translation analytic sense of Herbst and Howland, exist for the one dimensional Stark Hamiltonian, $-d^2/dx^2+q(x)+\varepsilon x$, with $q(x)$ a trigonometric polynomial, provided ε is sufficiently large.

1. Introduction

The problem of describing the motion of a particle in a one dimensional periodic solid pervaded by a uniform electric field has received considerable attention in the physics literature (see, e.g. [1,10,11]). Controversy has centered about the existence of so-called Wannier states, or Stark ladder resonances, which were described by Wannier in [9]. The purpose of this paper is to prove that for periodic potentials given by trigonometric polynomials resonances in the translation analytic sense of Herbst and Howland [5] exist for large values of the electric field.

The Hamiltonian for the system in question is

$$H(\varepsilon)=-\frac{d^2}{dx^2}+\varepsilon x+q(x),$$

acting in $L^2(\mathbb{R})$, where $q(x)=\sum_n c_n e^{inx}$ is a real valued trigonometric polynomial. Here ε is the strength of the electric field. It is known that for $\varepsilon\neq 0, \sigma(H(\varepsilon))=\mathbb{R}$ and is purely absolutely continuous [3,4]. To describe translation analyticity we begin with the unitary group of translations

$$(T(a)f)(x)=f(x+a) \tag{1.1}$$

for $a\in\mathbb{R}$ and note that

$$T(-a)H(\varepsilon)T(a)=-\frac{d^2}{dx^2}+\varepsilon x+q(x-a)-\varepsilon a.$$

Since $q(x)$ has an analytic extension to complex x we can define the complex

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