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Reduction Techniques for Infinite-Dimensional Hamiltonian Systems: Some Ideas and Applications*

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Abstract. In the language of tensor analysis on differentiable manifolds, we present a reduction method of integrability structures, and apply it to recover some well-known hierarchies of integrable nonlinear evolution equations.

1. Introduction

In the recent past, starting from the basic works of Lax [1] and of Gardner et al. [2], a lot of remarkable papers have been published which aimed at elucidating the integrability properties of some special classes of infinite-dimensional Hamiltonian systems, expressed by nonlinear evolution equations (NEE's). To describe the properties of such systems, different approaches have been followed: some of them were global, like the inverse scattering method in its various versions [3, 4], which for instance allows one to linearize the associated Cauchy problem and to construct relevant classes of explicit solutions. Some others were local, aiming at achieving an algebraic formulation of the integrability structure of those systems: among them, of prominent importance in our opinion is the one which can be associated with the names of Gel'fand-Dikii et al. [5], where the Hamiltonian structures supported by such systems are obtained from the dual algebra of certain infinite-dimensional Lie algebras, the algebras of pseudo-differential operators of negative degree.

The approach we wish to propose here does not belong to either of the families we have (indeed quite roughly) indicated above. It is in fact of a fairly geometrical nature: it investigates directly the integrability structures defined on some differentiable manifolds, and gives criteria which guarantee the reducibility of such structures on certain regular submanifolds. As special cases, through this approach one is able to recover the integrability structure of the more relevant hierarchies of NEE's studied in the literature. A systematic and exhaustive exposition of this method is given elsewhere [6]; however, it might be worthwhile to emphasize here some of its advantages. First of all, its tensor nature ensures that

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