Commun. Math. Phys. 99, 75–102 (1985)

Communications in Mathematical Physics

Spaces of Regular Gauge Field Configurations on a Lattice and Gauge Fixing Conditions

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Abstract. We consider spaces of lattice gauge field configurations satisfying gauge invariant regularity conditions, and intersections of these spaces with a surface given by gauge fixing conditions. We prove that if these conditions are chosen properly then configurations belonging to the intersection are small and regular.

In this paper we continue our investigation of the renormalization group method for lattice approximations of gauge field theories. A notion of regularity and spaces of regular gauge field configurations have appeared already in a natural way in [3]. There the compositions of averaging operations were considered and we have proved that they are regular (analytic) functions of the configurations U if the following regularity conditions are satisfied

$$|U(\partial p)-1|<\alpha_0\eta^2, \qquad \eta=L^{-k} \tag{0.1}$$

for α_0 sufficiently small and for plaquettes p contained in a subdomain $\Omega\subset\eta Z^d$. We refer the reader to [1, 3] for an explanation of notations and notions used here. We will use also almost all the results of those papers.

The regularity conditions (0.1) are the most fundamental conditions we impose on gauge field configurations. They are invariant with respect to gauge transformations

$$U(x,x')\to U^u(x,x')=u(x)U(x,x')u^{-1}(x'), \tag{0.2}$$

so the space of configurations satisfying (0.1) is decomposed into a union of orbits determined by the group of all gauge transformations.

In this paper we will consider more complicated regularity conditions, which are gauge invariant also; but let us explain our problem and results on the example (0.1).

Our renormalization group procedure will be based on solutions of some variational problems. To consider these problems we will have to fix gauge

* Research supported in part by the National Science Foundation under Grant PHY-82-03669