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Spins and Fermions on Arbitrary Lattices

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Abstract. It is proved that the system of free fermions on arbitrary lattices is equivalent to the set of locally interacting constrained spins. The fermionic counterpart of the unconstrained spin system is also derived. The generalization to the interacting theories is possible.

1. Introduction

In spite of many impressive results [1, 2], the Monte Carlo calculation with fermions on the lattice is still limited by the non-local nature of the problem. Mapping the fermionic into bosonic degrees of freedom may considerably change this situation. There are many attempts to do that, see for example [3], where you can also find a short historical outline and more references concerning this topic.

In this paper we will discuss the generalization of the Jordan-Wigner transformation to arbitrary dimensions [4]. We will prove the equivalence between the system of fermions and the set of locally interacting constrained spins. Such an equivalence was only conjectured in [4].

To begin with, we recall the construction proposed in [4]. Sections 2 and 3 contain necessary definitions and the proof of the equivalence. In Sect. 4 the role of the constraints is clarified. The fermionic counterpart of the unconstrained spin system is also derived there.

Consider the following Hamiltonian in two dimensions:

$$H = ia \sum_{n,e} \Phi^\dagger(n)\Phi(n+e) - \Phi^\dagger(n+e)\Phi(n),$$

where $n=(n_X, n_Y)$ labels the lattice sites and the unit vector $e=e_X, e_Y$. The fermion field Φ satisfies

$$\{\Phi^\dagger(m), \Phi(n)\} = 2\delta_{mn},$$

and other anticommutators vanish. Our aim is to express H in terms of the operators which rather commute at large distances, and hence, resemble the spin