Commun. Math. Phys. 98, 273–288 (1985)

Communications in
Mathematical
Physics
© Springer-Verlag 1985

Bounds on Completely Convergent Euclidean Feynman Graphs

J. Feldman*, J. Magnen, V. Rivasseau, and R. Sénéor

Centre de Physique Théorique de l'Ecole Polytechnique, Plateau de Palaiseau, F-91128 Palaiseau, Cedex, France

Abstract. Let G be a Euclidean Feynman graph containing $L(G)$ lines. We prove that if G has massive propagators and does not contain any divergent subgraphs its value is bounded by $K^{L(G)}$. We also prove the infrared analogue of this bound.

I. Introduction

This paper gives a very flexible technique for bounding Euclidean Feynman graphs. Such bounds are crucial not only in directly perturbation theoretic results [dCR, R, 'tH] but more generally in the proofs of convergence of many expansions in quantum field theory. (In fact, it was the demands of [FMRS 1] that led us to this work.) Typically, you need to estimate the rate of growth of the value of graphs as a function of their size, say as measured by the number of lines in the graph. In superrenormalizable models one can prove relatively easily [G 1, RS] that

$$|G| \leqq K^{L(G)}, \tag{1.1}$$

where $|G|$ is the value of the graph G, K is a constant, and $L(G)$ is the number of lines in G. However, in strictly renormalizable models the bound (1.1) is no longer true. Explicit families of graphs with factorial growth have been constructed [L, dCR]. These factorials arise from the renormalization subtractions. In [dCR] it is proven that, in ϕ_4^4, graphs that do not contain divergent subgraphs obey the bound (1.1). 't Hooft ['tH] has proven a similar result for planar ϕ_4^4.

In this paper we generalize this result to pretty well any model. In Sect. II we consider the ultraviolet case. In Sect. III we consider the infrared case. This case is also covered by [dCPR]. In the appendix we consider the case of only "logarithmically" convergent graphs.

We use a phase space expansion. In fact, because there is not very much obscuring detail it is one of the simplest applications of a phase space expansion we

* Permanent address: Department of Mathematics, University of British Columbia, Vancouver B.C., Canada V6T 1Y4