

Renormalization of the Higgs Model: Minimizers, Propagators and the Stability of Mean Field Theory★

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Dedicated to the memory of Kurt Symanzik

Abstract. We study the effective actions $S^{(k)}$ obtained by k iterations of a renormalization transformation of the U(1) Higgs model in $d=2$ or 3 space-time dimensions. We identify a quadratic approximation $S_Q^{(k)}$ to $S^{(k)}$ which we call mean field theory, and which will serve as the starting point for a convergent expansion of the Green's functions, uniformly in the lattice spacing. Here we show how the approximations $S_Q^{(k)}$ arise and how to handle gauge fixing, necessary for the analysis of the continuum limit. We also establish stability bounds on $S_Q^{(k)}$, uniformly in k . This is an essential step toward proving the existence of a gap in the mass spectrum and exponential decay of gauge invariant correlations.

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