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Convergent Perturbation Expansions for Euclidean Quantum Field Theory

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Dedicated to the memory of Kurt Symanzik

Abstract. Mayer perturbation theory is designed to provide computable convergent expansions which permit calculation of Greens functions in Euclidean quantum field theory to arbitrary accuracy, including "nonperturbative" contributions from large field fluctuations. Here we describe the expansions at the example of 3-dimensional $\lambda\phi^4$-theory (in continuous space). They are not essentially more complicated than standard perturbation theory. The n^{th} order term is expressed in terms of $O(n)$-dimensional integrals, and is of order λ^k if $4k-3 \leqq n \leqq 4k$.

1. Introduction

Mayer perturbation theory is designed to provide computable convergent expansions which permit calculation of Greens functions in Euclidean quantum field theory to arbitrary accuracy, including "nonperturbative contributions" from large field fluctuations. Their n^{th} order term is given by $O(n)$-dimensional integrals, as is the case in standard perturbation theory. In principle such expansions have a chance of converging for asymptotically free theories – including superrenormalizable ones – for problems where a small coupling constant is effective. Some models will require a more sophisticated treatment of the large field region, though. In this paper we describe the expansions in their simplest form at the example of weakly coupled massive $\lambda\phi^4$-theory in $\nu=3$ dimensions. Part I of the paper presents the main ideas and constructions. It describes the expansions in an elementary way, and discusses the relation to standard perturbation theory: The sum of all terms in the Mayer expansion up to order $4n$ equals the sum of all renormalized Feynman diagrams up to order n, plus a computable correction of higher order or nonperturbative origin. The raison d'être of Mayer expansions is their computability and convergence. Here we concentrate on computability. We plan to present estimates and discuss convergence properties in a subsequent article (Part II).

Mayer expansions [i.e., iterative solutions of Kirkwood Salsburg or Mayer Montroll equations, in place of Schwinger Dyson equations] were introduced into