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Large Deviations for Stationary Gaussian Processes*

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Abstract. In their previous work on large deviations the authors always assumed the base process to be Markovian whereas here they consider the base process to be stationary Gaussian. Similar large deviation results are obtained under natural hypotheses on the spectral density function of the base process. A rather explicit formula for the entropy involved is also obtained.

1. Introduction

Let $\{X_k\}$, $-\infty<k<\infty$, be a stationary Gaussian process with $E\{X_k\}=0$ and $E\{X_0X_j\}=\varrho_j=\frac{1}{2\pi}\int_0^{2\pi} e^{ij\theta}f(\theta)d\theta$. We assume that the spectral density function $f(\theta)$ is continuous on $[0,2\pi]$, $f(0)=f(2\pi)$, and

$$\int_0^{2\pi} \log f(\theta)d\theta > -\infty\,. \tag{1.1}$$

Let $\Omega=\prod_{j=-\infty}^{\infty} \mathbb{R}_j$ where, for each j, $\mathbb{R}_j$ is the real line, i.e., Ω is the space of doubly infinite sequences of real numbers. We specify a point $\omega\in\Omega$ by $\omega=\{x_k\}$, $-\infty<k<\infty$, and let $\omega(j)=x_j$ for $-\infty<j<\infty$. The process $\{X_k\}$ induces a probability measure P on Ω. We will denote integration over Ω with respect to P measure by $E^P\{\ \}$.

For each positive integer n and each $\omega\in\Omega$, let $\omega^{(n)}$ be the point in Ω obtained by the periodic extension in both directions of the elements $x_1,x_2,\dots,x_n$ of ω, i.e., if $\omega=\{x_k\}$, $-\infty<k<\infty$, then $\omega^{(n)}$ is the point

$$\dots,x_1,\dots,x_{n-1},x_n,x_1,x_2,\dots,x_n,x_1,x_2,\dots,x_n,\dots\,.$$

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