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The Energy-Momentum Spectrum in Local Field Theories with Broken Lorentz-Symmetry

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Abstract. Assuming locality of the observables and positivity of the energy it is shown that the joint spectrum of the energy-momentum operators has a Lorentz-invariant lower boundary in all superselection sectors. This result is of interest if the Lorentz-symmetry is (spontaneously) broken, such as in the charged sectors of quantum electrodynamics.

1. Introduction

The familiar theoretical explanation of the fact that the energy p_0 and the momentum $\mathbf{p}$ of an elementary particle are related by the fundamental equation $p_0 = (\mathbf{p}^2 + m^2)^{1/2}$, m being the mass of the particle, is based on the assumption that particle states can be described by vectors in some irreducible representation of the Poincaré group [1]. Alternatively, this form of the energy-momentum spectrum can be deduced from the hypothesis that the vectors describing particles can be generated from a vacuum vector with the help of local field operators [2]. The existence of Lorentz-transformations is not needed in the latter argument.

Both of these explanations are based on the (implicit) idea that, disregarding small "tails," particle states can be localized in bounded regions of space. Yet this picture is not correct in the case of particles carrying an electric charge. Since the electric charge is the source of the long-range Coulomb field, such states have poor localization properties which are incompatible with the existence of a continuous, unitary representation of the Lorentz group in the corresponding superselection sectors [3]. (Note that the geometrical action of a Lorentz transformation grows with the distance from the origin. So its effect on dislocalized states is large.) By the same token, there cannot exist local field operators generating these states from the vacuum [4]. Hence the conventional arguments establishing the Lorentz-invariance of the energy-momentum spectrum are not applicable in this case.

In the present article we reconsider the problem of the shape of the energy-momentum spectrum in the general framework of local quantum theory [5]. This