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Self-Avoiding Walk in 5 or More Dimensions*

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Abstract. Using an expansion based on the renormalization group philosophy we prove that for a T step weakly self-avoiding random walk in five or more dimensions the variance of the endpoint is of order T and the scaling limit is gaussian, as $T\to\infty$.

1. Introduction and Results

We consider walks $\omega(s)$ in $\mathbb{Z}^d$ which start at the origin and consist of $|\omega|=T$ nearest neighbor steps. If each such walk ω is assigned a weight proportional to

$$P_T(\omega)\equiv \prod_{0<s<t\leqq T}(1-\lambda\delta(\omega(s)-\omega(t))), \qquad 0<\lambda\leqq 1,$$

we say that the walk is weakly self-avoiding or self-repelling. Here s, t denote non-negative integers and $\delta(j)=1$ if $j=0$ and $\delta(j)=0$ otherwise. When $\lambda=1$ only walks which strictly self-avoid are counted. Now let us define an expectation of a functional F on paths ω, $|\omega|=T$ by

$$\langle F(\cdot)\rangle_T(\lambda)\equiv\frac{\sum_{|\omega|=T}F(\omega)\,P_T(\omega)}{\sum_{|\omega|=T}P_T(\omega)}.$$

A natural quantity to study is the mean square displacement of $\omega(T)$ defined by

$$R^2(T)\equiv\langle\omega^2(T)\rangle_T(\lambda).$$

In the physics literature $R^2(T)$ is expressed in terms of a critical exponent ν via the relation $R^2(T)\cong C(\lambda)\,T^{2\nu}$ for large T. On the basis of renormalization group

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