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On-shell Improved Lattice Gauge Theories

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Abstract. By means of a spectrum conserving transformation, we show that one of the 3 coefficients in Symanzik's improved action can be chosen freely, if only spectral quantities (masses of stable particles, heavy quark potential etc.) are to be improved. In perturbation theory, the other 2 coefficients are however completely determined and their values are obtained to lowest order.

1. Introduction

Symanzik's improvement programme [1–6] for lattice gauge theories (and other lattice theories) is designed to systematically reduce the cutoff dependence of on- and off-shell amplitudes near the continuum limit. Mainly, this is achieved by choosing an improved lattice action, which, at the first level of improvement, is equal to the standard one-plaquette action plus a linear combination of 3 operators[1] of dimension 6 with perturbatively calculable coefficients $c_i(g_0^2)$ (g_0: bare coupling constant, $i=1, 2, 3$). In addition, gauge invariant lattice operators are in general also *intrinsically* cutoff dependent and, in order to obtain improved correlation functions, must therefore be corrected by subtracting a combination of higher dimensional operators. The necessity of such subtractions has been explicitly demonstrated by Symanzik in the case of the non-linear σ-model [3. Sect. 4].

For the computation of the coefficients $c_i(g_0^2)$, the intrinsic cutoff dependence of operators is a potential source of difficulty, because it must be carefully disentangled from the "dynamical" cutoff effects, which are to be cancelled by improving the action. This problem can however be entirely avoided, if only the improvement of spectral quantities (e.g. the static quark-antiquark potential at physical distances) is required. Such quantities are independent of the choice of "interpolating" operator field and it therefore makes no difference whether the operators one uses have or have not been corrected.

In this paper, we show that the requirement of on-shell improvement places only two constraints on the coefficients $c_i(g_0^2)$ so that without loss one may choose $c_3(g_0^2)=0$, for example. $c_1(g_0^2)$ and $c_2(g_0^2)$ are then completely fixed and can be

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1 By abuse of notation, we use the word "operator" for any Euclidean ($\mathbb{C}$-number) field, which can be composed from the fundamental gauge field