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Monte Carlo Simulations for Quantum Field Theories Involving Fermions

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Abstract. We present a new variant of a Monte Carlo procedure for euclidean quantum field theories with fermions. On a lattice every term contributing to the expansion of the fermion determinant is interpreted as a configuration of self-avoiding oriented closed loops which represent the fermionic vacuum fluctuations. These loops are related to Symanzik's polymer description of euclidean quantum field theory. The method is extended to the determination of fermionic Green's functions. We test our method on the Scalapino-Sugar model in one, two, three, and four dimensions. Good agreement with exactly known results is found.

1. Introduction

In recent years Monte Carlo simulations for euclidean lattice models have been of considerable help in improving our understanding of those relativistic quantum field theories, which are supposed to describe high energy particle physics. This includes in particular gauge theories. Now any realistic model for particle interactions includes fermionic fields like quark fields. It is, therefore, important to simulate systems with fermionic degrees of freedom. There have been several proposals to deal with this problem, see e.g. [1–16], or [17–19] for a review. However, all these methods require extensive computing time and some of them only work for two-dimensional models or are only approximations from the beginning: quenched approximation, hopping parameter expansion etc. In particular for the interesting case of four-dimensional lattices no way has yet been found to perform Monte Carlo calculations including fermions as efficiently as they can be done when bosonic fields only are present.

It is the aim of this paper to propose a new numerical method which basically treats all fields on the same footing during the upgrading procedure. This new way of treating fermions applies to all lattice models known to the authors and may

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